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Intrinsic vanishing of energy and momenta in a universe

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Abstract We present a new approach to the question of properly defining energy and momenta for non asymptotically Minkowskian spaces in General Relativity, in the case where these energy and momenta are conserved. In order to do this, we first prove that there always exist some special Gauss coordinates for which the conserved linear and angular 3-momenta intrinsically vanish. This allows us to consider the case of *creatable* universes (the universes whose proper 4-momenta vanish) in a consistent way, which is the main interest of the paper. When applied to the Friedmann-Lemaître-Robertson-Walker case, perturbed or not, our formalism leads to previous results, according to most literature on the subject. Some future work that should be done is mentioned.

Keywords Energy and momenta of the Universe · Non asymptotic flatness · Intrinsic vanishing of momenta

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1 Introduction

1.1 General considerations

In General Relativity, the problem of associating linear and angular 4-momenta to a finite space-time region and the related idea to define (for a general space-time) such global quantities have been approached from different, but

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not necessarily contradictory, perspectives. See [1] for an extensive and critical review on the current status of the problem.

Diverse mathematical objects (energy-momentum pseudotensors and superpotentials, flat or curved background metrics, Killing vectors or other fields generating generalized symmetries, etc.) and several geometrical techniques (3+1 or 2+2 space-time splittings, initial data constraints and boundary conditions, etc.) seem appropriate to deal with this issue. See, for example, [2, 3, 4, 5, 6] for some detailed explanations and general comments on these subjects.

However, nowadays, no consensus on a preferred approach nor any complete or definitive answer to the problem of how to associate linear and angular 4-momenta to a general space-time seem to have been reached by the relativistic community. Of course, the existing points of view don't exclude each other and seem to point towards the correct understanding of the problem, while the possibility of new approaches remains still open.

Nevertheless, the reader should be warned about the presence of a lot of criticisms in the current literature to the pseudotensor approach to define the 4-momenta of a physical space-time, which is the approach adopted in the present paper. These criticisms stress that the approach is by no means a covariant one (see for example [7,8]), or argue against any definition of energy in General Relativity referring to 3-surface integrals, instead of being quasi-local (referring to 2-surface integrals) from the very beginning [9], or even accept the approach for asymptotically flat space-times but express some doubts for the non asymptotically flat ones [10].

Although the covariant approach to the definition of quasi-local conserved quantities in General Relativity followed in [7,8,9] and the results obtained seem very interesting, we cannot fully share those criticisms.

Before giving our personal opinion about it, let us begin remarking that in [11] a particular "covariant Hamiltonian approach to quasi-local energy" is presented. In this approach, each pseudotensor corresponds to a Hamiltonian boundary term, which brings the authors to the conclusion that "Hamiltonian approach to quasi-local energy-momentum rehabilitates the pseudotensors". Quoting this conclusion, Vargas [12] used again the pseudotensorial method to calculate the energy of the universe in teleparallel gravity. The conclusion was quoted in [13] too.

Regarding the objection prescribing a quasi-local definition of energy in General Relativity [9], we recall that our pseudotensorial method yields 4-momenta that are quasi-local quantities, in the sense that they can be expressed subsequently as 2-surface integrals, even if their original definitions were through 3-surface integrals.

As far as the remarked [7,8] non covariance of the pseudotensorial method is concerned, we recall that there is nothing invalidating in the fact that the energy of a physical system can depend on the reference frame used and that, at the same time, we look for some natural special frame in order to define some "proper" energy and momenta: at least, nothing, apparently, that from the very beginning prevents us from approaching this problem. The same frame dependence is present in classical mechanics, as it is mentioned in [14] in re-

lation to the general problem of defining energy in a covariant way. But, for a particle, for example, we can select a "natural" special frame (the one where the 3-momentum vanishes) to define the "proper" energy of the particle. In a similar way, but pointing to General Relativity, we must select a "natural" congruence of observers and a "natural" coordinate system related to it in order to get rid of the spurious energy and momenta associated to the fictitious gravitational field related to "bad" observers (for example, observers which do not fall freely) and "bad" related coordinates, so that we can reach some kind of space-time "proper" 4-momenta (see section 2, in the paragraph beginning just after Eqs. (1)-(4)). This is in fact what is performed in the non problematic case of asymptotically flat space-times, where the "proper" coordinates are those which are asymptotically Lorentzian, such that the corresponding energy is the "proper" energy. We can think about the pseudotensorial method developed in the present paper as a proper generalization of this procedure to the non asymptotically flat case, which would be a certain response to the aforementioned doubts in [10]. In fact, in [1] Szabados summarizes the question by simply saying: "to use the pseudotensors, a "natural" choice for a "preferred" coordinate system would be needed". This is just what we have done in the present paper.

1.2 Summary of some previous work

In a previous paper [15], we addressed the question of properly defining the linear and the angular 4-momenta of a significant family of non asymptotically flat space-times. As it is well known, and as we have just commented, see for example [16] or [17], this proper definition can be accomplished without difficulty in the opposite case of asymptotically flat space-times, but not in the general case (for a concise and readable account, see also [18]). The reason for this difficulty in the general case stays in the dramatic dependence of these momenta on the coordinate system used. This fact is very well known but very few times has properly been taken into account in the literature of the field, where some authors use a given coordinate system to calculate some of the momenta, without any comments on the rightness of the coordinate selection that has been done. For related questions on this subject see, for instance, [2, 19, 20, 21, 22] and references therein.

The family of space-times that we are going to consider in the present paper is the family of all non asymptotically flat space-times where these well defined momenta are conserved in time. We call these particular space-times *universes*, since it is to be expected that any space-time which could represent the actual universe should have conserved momenta, provided that these momenta be properly defined, which is the goal achieved in the present paper.

Then like in [15], we call *creatable universes* the universes which have vanishing 4-momenta, since again this is what could be expected to happen if the considered universe raised from a quantum fluctuation of the vacuum [23, 24]. In fact, the question of the *creatable universes* is our main motivation to

consider the subject of properly defining the momenta of non asymptotically flat space-times. Demanding the vanishing of the momenta can be a way of saying something relevant about how our actual Universe looks like either now or in the preinflationary phase. Thus, for example, in [25], perturbed flat Friedmann-Lemaître-Robertson-Walker (FLRW) universes according to standard inflation, and also the perturbed open universes, were found to be non creatable. Therefore, among the inflationary perturbed FLRW universes, only the closed ones would be left as good candidates to represent the actual Universe.

In the present paper we present a new approach to the subject of properly defining the two 4-momenta of a *universe*, as compared with the one presented in the above reference [15] whose results we summarize here:

In [15] we considered a given space-time, not necessarily asymptotically flat, with its general expressions for the linear 4-momentum and the angular 4-momentum obtained from the Weinberg complex. Then, we assumed that the "intrinsic" values of these 4-momenta are conserved, that is, the values corresponding to some "proper" coordinate system that has to be consequently determined. As mentioned above, space-times endowed with such conserved 4-momenta are called *universes* in the referred paper. We argued why these coordinates, $\{t, x^i\}$, are, to begin with, Gauss coordinates referred to some space-like 3-surface, Σ_3, whose equation then becomes $t = t_0$. We proved that the corresponding 3-space metric, dl_0^2, that is, $dl^2 = g_{ij}dx^i dx^j$ for $t = t_0$, is asymptotically conformally flat over the 2-surface boundary, Σ_2, of Σ_3, and we used 3-space coordinates x^i adapted to this circumstance, i.e., $dl^2|_{\Sigma_2} = f\delta_{ij}dx^i dx^j$, with f some function defined on Σ_2. Finally, looking for universes with vanishing 4-momenta, we assumed that the metric components g_{ij} go to zero fast enough when we approach Σ_2. In this way, we were able to define a family of universes whose 4-momenta vanish irrespective of the selected Σ_3 and of the conformal coordinates used in the corresponding boundary Σ_2. The family covers in particular the FLRW universes, for which we obtain the 4-momenta values previously obtained by some authors but not by all of them (see section 6 for some comments about these agreements and disagreements).

The new approach is by no means a minor variation of the ancient one, as we explain in the next subsection:

1.3 Outline of the paper

In the present paper, given a *universe*, when trying to select the appropriate coordinate systems in order to properly define its two 4-momenta, P^α and $J^{\alpha\beta}$, we impose alternatively to [15] that both 3-momenta, P^i and J^{ij}, vanish, the last one irrespective of the origin of momentum. However, according to what we have just explained about [15], we rest on Gauss coordinates based on some space-like 3-surface, Σ_3, such that the corresponding 3-space metric can be written in a conformally flat way on the boundary of Σ_3. Such Gauss coordinate systems, where both 3-momenta vanish (the last one irrespective

of the origin), which at the same time are coordinates satisfying the above conformally flat property, will be called here *intrinsic* coordinate systems. Obviously, we first prove here that these *intrinsic* coordinate systems always exist for any *universe*, which is a capital new result.

However, in [15], in order to have vanishing 4-momenta, we had to assume that the metric and its first derivatives went fast enough to zero when we approach the boundary of Σ_3. In the present paper we do not need to make such an *ad hoc* assumption, and so our present approach, as compared with the one in [15], stresses the *intrinsic* character, and so the physical meaning, of the given definition of the universe 4-momenta.

The paper is organized as follows: In Sect. 2, given a space-like 3-surface, Σ_3, we give the corresponding family of coordinate systems where to choose the right coordinate systems to properly define the linear and the angular 4-momenta associated to this Σ_3. In section 3, we consider all 3-surfaces Σ_3 showing the same boundary Σ_2. Then, by defining what we have called intrinsic coordinates, we select the 3-surfaces Σ_3 for which the linear and the angular 3-momenta vanish, after proving that this result is valid for some Σ_3. In Sect. 4, we define the notion of *creatable* universe and we discuss briefly its goodness. In Sect. 5, we invoke some previous results to check the *creatibility* of the perturbed FLRW models in the new scheme, reproducing the known conclusions also obtained in [25] on these models. Had we not been able to confirm these results in the present approach, we should consider them as actually non valid, since we find now that the vanishing of P^i and J^{ij} in the coordinates used is mandatory (although not sufficient) to confer physical meaning to the 4-momenta definition used. Finally, in Sect. 6, we comment on the, sometimes, different values of the 4-momenta for FLRW universes, found by different authors, including our work, and we point out which is, in our opinion, the main interest of the paper, and in relation to this we refer to some future work.

We still add three appendices where some calculations are given in detail.

A short report containing some results, without proof, of this work was presented at the Spanish Relativity Meeting ERE-2009 [26].

2 The energy and momenta of a *universe*, associated to a given space-like 3-surface

In order to define the linear and angular 4-momenta of a *universe* we will use the Weinberg complex [16].

It remains to be checked wether the final results obtained in the present paper keep still valid for other complexes that, like the Weinberg one, are symmetric in their two indices, which allows us to build the corresponding angular 4-momentum. This criterion leads to discard other pseudotensors as the ones by Einstein, Bergmann or Møller, but not by Papapetrou or Landau-Lifshitz. Any case, the Weinberg complex is a very natural one, as it is very convincintly argued in [16], letting aside the interesting well known fact that

Weinberg complex gives also the correct 4-momenta in a Schwarzschild metric. As far as the Golberg pseudotensor is concerned, it is a very general one to which the Weinberg one belongs as a particular case. About these different pseudotensor see, for example [11].

Before going to the notion of the 4-momenta of a general space-time, some previous definitions and considerations.

Metric signature: we use signature +2, that is, $d\tau^2 \equiv -ds^2 = -g_{\alpha\beta}dx^\alpha dx^\beta$ is the square of the corresponding elementary proper time when $ds^2 < 0$. Thus, Greek indices take values from 0 to 3, and Latin indices from 1 to 3.

Gauss coordinates: we can define them as coordinates in which $ds^2 = -dt^2 + dl^2$ with $dl^2 \equiv g_{ij}dx^i dx^j$ being positive defined. Although not globally, we can build such a coordinate system by referring the space-time metric to a congruence of observers that fall freely, by endowing each observer with a canonical (physical) clock, and finally by *synchronizing* (see next the notion of synchronization) all these different clocks [27]. In these coordinates, the 3-surface $t = t_0$, with t_0 any constant time, is a space-like 3-surface, Σ_3, orthogonal to the congruence, in whose neighborhood the Gauss coordinate system is defined.

Clock synchronization: given such a congruence of observers, each one endowed with his canonical clock, we can synchronize all them with the same method used in special relativity (using come and back light beams: see again [27]). Then, one finds that all events belonging to Σ_3, that is, all the events $t = t_0$, are simultaneous according to this definition. Thus, t is a physical and universal time (like time in the Minkowski space is, for example).

Then, to properly define the notion of 4-momenta of a *universe*, associated to some space-like 3-surface, Σ_3, we will take Gauss coordinates associated to this 3-surface, Σ_3, in the neighborhood of it (we explain next why we make this choice). In the Weinberg approach [16], the linear and angular momenta of the gravitational field are incidentally defined by integrating on Σ_3. The main Weinberg pursuit is to obtain an integral balance relation for each momentum component such that, as it is standard, the time derivative of a 3-volume Σ_3 integral for this component density equates (using Gauss theorem) the minus correspondent flux through the 2-surface boundary Σ_2 of the 3-volume Σ_3. These integrated balance equations come straightly from the vanishing of the ordinary (not covariant) divergence of the pseudotensor plus the energy-momentum tensor and, by construction, the 3-volume integrals incorporate a non geometric volume element. That is, this 3-volume element is just $dx^1dx^2dx^3$ independently of the meaning of this coordinates (see the details in [16]). As a result, these volume integrals can be expected to have a physical meaning only for some kinds of physical coordinates. On the other hand, these 3-volume integrals, using again Gauss theorem, can be written as 2-surface integrals over the 3-volume boundary, Σ_2. Then, according to [16], we have for the corresponding energy, P^0, linear 3-momentum, P^i, angular 3-momentum, J^{ij}, and components J^{0i} of the angular 4-momentum, of the

universe:

$$P^0 = \kappa \int (\partial_j g_{ij} - \partial_i g) d\Sigma_{2i}, \tag{1}$$

$$P^i = \kappa \int (\partial_0 g \delta_{ij} - \partial_0 g_{ij}) d\Sigma_{2j}, \tag{2}$$

$$J^{jk} = \kappa \int (x_k \partial_0 g_{ij} - x_j \partial_0 g_{ki}) d\Sigma_{2i}, \tag{3}$$

$$J^{0i} = P^i t - \kappa \int [(\partial_k g_{kj} - \partial_j g) x_i + g\delta_{ij} - g_{ij}] d\Sigma_{2j}, \tag{4}$$

where we have used the following notation: $\kappa^{-1} \equiv 16\pi G$, G is the Newton constant and we have taken $c = 1$ for the speed of light, $g \equiv \delta^{ij} g_{ij}$, ∂_0 is the partial derivative with respect to $x^0 \equiv t$, and $d\Sigma_{2i}$ is the surface element of Σ_2, the boundary of Σ_3. Further, indices $i, j, ...$ are raised or lowered with the Kronecker δ and angular momentum has been taken with respect to the origin of coordinates.[1]

Why Gauss coordinates? We expect any well behaved universe, V_4, to have well defined energy and momenta, i. e., P^α and $J^{\alpha\beta}$, $\alpha, \beta, ... = 0, 1, 2, 3$, such that they are finite and conserved in time (a *universe* in our notation). So, for this conservation to make physical sense, we need to use a *physical* and *universal* time, according to the definition introduced at the beginning of the present Section. Then, still in accordance with these definitions, we are conveyed to use a Gauss coordinate system with its universal time to properly define the *universe* 4-momenta. Moreover, using this Gauss time, any component of the 4-momenta appears as the addition (the 2-surface integral) of the corresponding *simultaneous* densities, as it must be from a physical point of view.

Then, as defined above, we will have for the line element of V_4:

$$ds^2 = -dt^2 + dl^2, \quad dl^2 = g_{ij} dx^i dx^j, \tag{5}$$

and we can write $t = t_0 = \textit{constant}$ for the equation of Σ_3.

The area of the 2-surface boundary Σ_2 could be zero, finite or infinite. Let us precise that in the first case, when the area is zero, the 4-momenta do not necessarily vanish, unless the metric and its first derivatives remain conveniently bounded when we approach Σ_2.

Obviously, we have as many local families of Gauss coordinates as space-like 3-surfaces, Σ_3, we have in V_4. Then, P^α and $J^{\alpha\beta}$ will depend on Σ_3, which is not a drawback in itself (the energy of a physical system in the Minkowski space-time also depends on the Σ_3 chosen, i.e., on the Lorentzian coordinates

[1] In the Weinberg book [16], the case of an asymptotically flat space-time is the only considered. Nevertheless, it is straightforward to see that the displayed treatment also covers the case of non asymptotically flat ones, provided that P^α and $J^{\alpha\beta}$ as defined in (1)-(4) exist, i.e., provided that the corresponding integrals converge.

chosen). But the problem is that, given a space-like 3-surface, Σ_3, we can still have many different 4-momenta, according to the particular Gauss coordinate we choose, associated to the same Σ_3.

Let us begin suppressing a part of the arbitrariness left in the choice of Gauss coordinates. In order to do this, we will choose Gauss coordinates such that the equation of Σ_2 becomes $x^3 = 0$, and dl^2 on Σ_2 reads

$$dl^2(t = t_0, x^3 = 0) \equiv dl^2|_{\Sigma_2} = f(x^a)\delta_{ij}dx^i dx^j, \tag{6}$$

with f some given function,[2] $a, b, ... = 1, 2$, and furthermore

$$g_{3a}(t = t_0) = 0. \tag{7}$$

That can always be done (see [15] and the last paragraph of the present section). Therefore, the induced 3-volume element $dx^1dx^2dx^3$ used in our 3-volume integrals to define energy and momenta (see the just previous paragraph to Eqs. (1)-(4)) becomes physically sound.

Furthermore, since $t = t_0$, $x^3 = 0$, is now the equation of the 2-surface Σ_2, the expressions (1)-(4) for P^α and $J^{\alpha\beta}$ simplify to:

$$P^0 = -\kappa \int \partial_3 g_{aa} \; dx^1dx^2 \,, \tag{8}$$

$$P^a = -\kappa \int \partial_0 g_{3a} \; dx^1dx^2 \,, \tag{9}$$

$$P^3 = \kappa \int \partial_0 g_{aa} \; dx^1dx^2 \,, \tag{10}$$

$$J^{ij} = \kappa \int (x_j\partial_0 g_{3i} - x_i\partial_0 g_{3j}) \; dx^1dx^2 \,, \tag{11}$$

$$J^{0a} = P^a t_0 + \kappa \int x^a \partial_3 g_{bb} \; dx^1dx^2 \,, \tag{12}$$

$$J^{03} = P^3 t_0 - \kappa \int g_{aa} \; dx^1dx^2 \tag{13}$$

where $g_{aa} = g_{11} + g_{22}$. Notice that, since the 3-volume element was $dx^1dx^2dx^3$, corresponding to the 3-metric $\delta_{ij}dx^idx^j$, and since Σ_2 is $x^3 = 0$, the induced 2-metric on Σ_2 is $\delta_{ab}dx^adx^b$, whose metric determinant value is 1, and so the 2-surface element $d\Sigma_{2i}$ has become dx^1dx^2.

Let us point that Σ_2 could also be made of different sheets. Thus, in the above Gaussian coordinates, these sheets could be the six faces of a cube that increases without limit. Its corresponding six equations would be $\forall i, x^i = \pm L$, for $L \to \infty$. These equations could be written $x'^i = 0$, by defining the new coordinates $x'^i = x^i \mp L$ and putting $x'^i = \pm|\epsilon|$, with $L \to \infty$ and $\epsilon \to 0$, that is, we will first calculate the integrals (1)-(4) for finite values of L and $|\epsilon|$, and then we will take the above limits. The new coordinates x'^i are Gauss

[2] Expression (6) will not always be valid globally. In this case we will have to cover Σ_2 with different charts, performing each Σ_2–integration over each chart, and summing up the different non overlapping chart contributions.

coordinates with dl'^2 conformally flat on Σ_2 as it must be. All this means, in particular, that the right hand side of, for example, (8) would actually be the sum of six similar integrals, one for each cube face. Nevertheless, in case we would have taken $x^i = \pm L$, $L \to \infty$, as the equation of Σ_2, it can be easily seen that essentially nothing would change in the present paper.

3 Proving that, for any *universe*, intrinsic coordinates always exist

We start with a Gauss coordinate frame, $\{x^\alpha\}$, such that (6) and (7) are satisfied. Let us prove that, from this coordinate frame, we always can move to an *intrinsic* coordinate frame as defined in the Introduction. Let it be a coordinate transformation $x^\alpha \to x'^\alpha$ such that in the neighborhood of Σ_2 we can write the expansion in x'^3 and $t' - t_0$

$$\begin{aligned} t - t_0 &= {}_0\xi^1 x'^3 \ + \ {}_1\xi^0 \ (t' - t_0) + \dots \ , \\ x^3 \equiv x_3 &= {}_0\xi^1_3 x'^3 \ + \ {}_1\xi^0_3 \ (t' - t_0) + \dots \ , \\ x^a \equiv x_a &= {}_0\xi^0_a + \ {}_0\xi^1_a x'^3 \ + \ {}_1\xi^0_a \ (t' - t_0) + \dots \ , \end{aligned} \tag{14}$$

where the expansion coefficients ${}_n\xi^m$ and ${}_n\xi^m_i$, with $n, m = 0, 1, 2, \dots$, are functions of x'^a. Notice that this coordinate transformation is completely general except for the fact that

$${}_0\xi^0 = \ {}_0\xi^0_3 = 0. \tag{15}$$

To begin with, we will require that the new coordinates $\{x'^\alpha\}$ be Gauss coordinates for V_4, associated to the space-like 3-surface Σ'_3, i.e. to $t' = t_0$. Actually, we will only require that the $\{x'^\alpha\}$ be Gauss coordinates in the neighborhood of Σ'_2, the boundary of Σ'_3. Reducing our original requirement in this way is worth since it is known that Gaussian coordinates, sooner or later, develop singularities under appropriate physical conditions (focussing theorem, see for exemple [28]).

On the other hand, since the equation of the boundary Σ_2 is $t = t_0$, $x^3 = 0$, this means by definition of boundary that the metric, g_{ij}, and its first derivatives, all them for $t = t_0$, exist only for, let us say, $x^3 > 0$, at least in some elementary interval around $x^3 = 0$. Then, since

$$g'_{ij} = -\frac{\partial t}{\partial x'^i}\frac{\partial t}{\partial x'^j} + \frac{\partial x^k}{\partial x'^i}\frac{\partial x^l}{\partial x'^j} g_{lk} \tag{16}$$

Σ_2 will still be the boundary of Σ'_3, provided that the functions $x^\alpha(x'^\beta)$ and its derivatives, up to second order included, be well defined coordinates wherever the metric g_{ij} and its first derivatives are well defined in the neighborhood of Σ_2.

Notice that, from Eqs. (14), the equation of Σ_2 in the new coordinates $\{x'^\alpha\}$ reads $t' = t_0, x'^3 = 0$. Thus, if we name Σ'_2 the 2-surface $t' = t_0$, $x'^3 = 0$, we can say that $\Sigma'_2 = \Sigma_2$.

Then, besides requiring that $\{x'^\alpha\}$ be Gauss coordinates for V_4 in the neighborhood of Σ_2, the boundary of Σ'_3, we will require that, according to (6),

$$dl'^2(t=t_0, x^3=0) \equiv dl'^2|_{\Sigma_2} = f'(x'^a)\delta_{ij}dx'^i dx'^j. \tag{17}$$

Furthermore, we will still require that the new linear and angular 3-momenta, P'^i and J'^{ij} (see (9), (10) and (11)), vanish, the last one irrespective of the origin. That is to say, we want the new coordinate system $\{x'^\alpha\}$ to be an *intrinsic* coordinate system as defined in the Introduction.

From Eq. (11) we can see very easily that a necessary and sufficient condition to have $J^{ij}=0$, irrespective of the momentum origin, is that

$$\int \partial_0 g_{3i}\ dx^1 dx^2 = 0, \quad \forall i, \tag{18}$$

which for $i=a$ leads to $P^a=0$. On the other hand, the three components of J^{ij} can be more explicitly written

$$J^{12} = \kappa \int (x^2 \partial_0 g_{31} - x^1 \partial_0 g_{32})\ dx^1 dx^2, \tag{19}$$

$$J^{3a} = \kappa \int x^a \partial_0 g_{33}\ dx^1 dx^2. \tag{20}$$

Then, aside (19) and (20) we also have (18). A sufficient condition to have all this at the same time is that the g_{3i} metric components be such that

$$\int \partial_0 g_{33}\ dx^1 = \int \partial_0 g_{33}\ dx^2 = 0, \tag{21}$$

$$\int \partial_0 g_{3a}\ dx^{(a)} = 0, \tag{22}$$

where putting the a-index between parenthesis means that the index is not summed up.

In all: we start from a coordinate system, $\{x^\alpha\}$, where we have

$$g_{00} = -1, \quad g_{0i} = 0, \tag{23}$$

$$g_{3a}(t=t_0) = 0, \quad g_{ij}(t=t_0, x^3=0) = f(x^a)\delta_{ij}, \tag{24}$$

and we want to prove that a coordinate transformation (14) exists such that the new components of the metric satisfy

$$g'_{00} = -1, \quad g'_{0i} = 0, \tag{25}$$

$$g'_{ij}(t'=t_0, x'^3=0) = f'(x'^a)\delta_{ij}, \tag{26}$$

and that, according to (9), (10), (18), (19) and (20), we have:

$$\int \partial'_0 g'_{aa}\ dx'^1 dx'^2 = 0, \ \int \partial'_0 g'_{3i}\ dx'^1 dx'^2 = 0, \tag{27}$$

$$\int (x'^2 \partial'_0 g'_{31} - x'^1 \partial'_0 g'_{32}) \ dx'^1 dx'^2 = 0, \tag{28}$$

$$\int x'^a \partial'_0 g'_{33} \ dx'^1 dx'^2 = 0, \tag{29}$$

where ∂'_0 means time derivative with respect the new time t'.

What all these conditions (25)-(29) say about the functions ${}_n\xi^m$ and ${}_n\xi^m_i$ which are present in the coordinate transformation (14)?

In order to answer this question let us first write in the neighborhood of Σ_2:

$$g_{ij} = {}_0g^0_{ij} + {}_0g^1_{ij} x^3 + {}_1g^0_{ij}(t - t_0) + ..., \tag{30}$$

where, according to the notation used in (14), we have:

$$ {}_0g^0_{ij} = g_{ij}(t = t_0, x^3 = 0), \tag{31}$$

$$ {}_0g^1_{ij} = \partial_3 g_{ij}(t = t_0, x^3 = 0), \tag{32}$$

$$ {}_1g^0_{ij} = \partial_0 g_{ij}(t = t_0, x^3 = 0), \tag{33}$$

and so on. This means that the expansion coefficients ${}_ng^m_{ij}$ in (30) are functions only of x^a.

Then, Eqs. (27), (28) and (29) read

$$\int {}_1g'^0_{aa} \ dx'^1 dx'^2 = 0, \ \int {}_1g'^0_{3i} \ dx'^1 dx'^2 = 0, \tag{34}$$

$$\int (x'^2 \ {}_1g'^0_{31} - x'^1 \ {}_1g'^0_{32}) \ dx'^1 dx'^2 = 0, \tag{35}$$

$$\int x'^a \ {}_1g'^0_{33} \ dx'^1 dx'^2 = 0, \tag{36}$$

where, similarly to (31), (32) and (33), we have put

$$ {}_1g'^0_{3a} = \partial'_0 g'_{3a}(t' = t_0, x'^3 = 0) = \partial'_0 g'_{3a}(t = t_0, x^3 = 0), \tag{37}$$

$$ {}_1g'^0_{33} = \partial'_0 g'_{33}(t = t_0, x^3 = 0), \tag{38}$$

since, according to (14), $t' = t_0, x'^3 = 0 \Leftrightarrow t = t_0, x^3 = 0$.

Similarly, Eq. (26) reads now:

$$ {}_0g'^0_{ij} = f'(x'^a)\delta_{ij}. \tag{39}$$

Thus, with the new notation ${}_ng'^m_{ij}$, the conditions (25)-(29) become (25), (34)-(36) and (39).

Let us first consider conditions (25). To zero order in t' and x'^3 (that is, strictly on the boundary Σ_2) these conditions become

$$({}_1\xi^0)^2 - f({}_1\xi^0_3)^2 = 1, \ \ {}_1\xi^0_a = 0, \ \ {}_1\xi^0 \ {}_0\xi^1 = f \ {}_1\xi^0_3 \ {}_0\xi^1_3, \tag{40}$$

from $g'_{00} = -1$, $g'_{0a} = 0$ and $g'_{03} = 0$, respectively.

On the other hand, conditions (39) become

$$f'\delta_{ab} = f\delta_{cd}\,\frac{\partial_0\xi^0_c}{\partial x'^a}\,\frac{\partial_0\xi^0_d}{\partial x'^b},\quad {}_0\xi^1_a = 0,\;\; f({}_0\xi^1_3)^2 - ({}_0\xi^1)^2 = f', \tag{41}$$

from ${}_0g'^0_{ab} = f'\delta_{ab}$, ${}_0g'^0_{3a} = 0$ and ${}_0g'^0_{33} = f'$, respectively.

It can be seen that the general solution of the system (40) and (41) is

$${}_1\xi^0_a = {}_0\xi^1_a = 0. \tag{42}$$

$${}_1\xi^0 = \sqrt{\frac{f}{f'}}\;{}_0\xi^1_3 = \cosh\psi, \tag{43}$$

$$\frac{1}{\sqrt{f'}}\;{}_0\xi^1 = \sqrt{f}\;{}_1\xi^0_3 = \sinh\psi, \tag{44}$$

plus

$$M_{ab} \equiv \frac{\partial_0\xi^0_a}{\partial x'^b} = \lambda\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix},\; \lambda \equiv \sqrt{f'/f}, \tag{45}$$

the Jacobian matrix of the conformal transformation in two dimensions. In (43), (44) and (45) the functions ψ, λ and θ are arbitrary functions of x'^a. Notice that (45) says that in the integrals (34)-(36) we can put $dx'^1dx'^2 = \lambda^{-2}dx^1dx^2$.

We still must have:

$$\begin{aligned}
{}_1g'^0_{3a} &= (f\;{}_1\xi^1_b + {}_1g^0_{3b}\;{}_1\xi^0\;{}_0\xi^1_3)M_{ba} + f\;{}_0\xi^1_3\;{}_1\xi^0_{3,a} - {}_0\xi^1\;{}_1\xi^0_{,a}\,, &(46)\\
{}_1g'^0_{33} &= 2(f\;{}_0\xi^1_3\;{}_1\xi^1_3 - {}_0\xi^1\;{}_1\xi^1) + {}_1g^0_{33}\;{}_1\xi^0({}_0\xi^1_3)^2\,, &(47)\\
{}_1g'^0_{aa} &= ({}_1g^0_{bc}\;{}_1\xi^0 + {}_0g^1_{bc}\;{}_1\xi^0_3)M_{ba}M_{ca} = \lambda^2({}_1g^0_{aa}\;{}_1\xi^0 + {}_0g^1_{aa}\;{}_1\xi^0_3)\,, &(48)
\end{aligned}$$

where ${}_1g'^0_{3a}$, ${}_1g'^0_{33}$ and ${}_1g'^0_{aa}$ are functions of x'^a such that (34), (35) and (36) are satisfied. The derivative with respect x'^a is denoted by $,a$ (for instance, ${}_1\xi^0_{3,a} \equiv \frac{\partial\;{}_1\xi^0_3}{\partial x'^a}$).

In Eqs. (46) and (47) new expansion coefficients ${}_1\xi^1_i$ and ${}_1\xi^1$ appear, which are not included in (42)-(45). But they appear in Eq. (25) when it is taken to zero order in t' and order one in x'^3 (remember that up to now we have only considered the lowest order of this equation), which becomes:

$$\begin{aligned}
{}_0g'^1_{0a} &= (f\;{}_1\xi^1_b + {}_1g^0_{3b}\;{}_0\xi^1\;{}_1\xi^0_3)M_{ba} + f\;{}_1\xi^0_3\;{}_0\xi^1_{3,a} - {}_1\xi^0\;{}_0\xi^1_{,a} = 0, &(49)\\
{}_0g'^1_{03} &= f({}_1\xi^0_3\;{}_0\xi^2_3 + {}_1\xi^1_3\;{}_0\xi^1_3) - {}_1\xi^0\;{}_0\xi^2 - {}_1\xi^1\;{}_0\xi^1 + {}_1g^0_{33}\;{}_0\xi^1\;{}_1\xi^0_3\;{}_0\xi^1_3 = 0, &(50)\\
{}_0g'^1_{00} &= 2(f\;{}_1\xi^0_3\;{}_1\xi^1_3 - {}_1\xi^0\;{}_1\xi^1) + {}_1g^0_{33}\;{}_0\xi^1({}_1\xi^0_3)^2 = 0. &(51)
\end{aligned}$$

Therefore, we must fit the new expansion coefficients, ${}_1\xi^1_i$ and ${}_1\xi^1$, plus the arbitrary functions λ, θ, and ψ, of Eqs. (43)-(45), in order to satisfy the system (46)-(48) plus (49)-(51). Let us show that this can always be done.

First, since the Jacobian matrix M_{ab} is regular, we can always fit the ${}_1\xi^1_b$ such that the two Eqs. (46) be satisfied. Second, since $f \neq 0$, (dl^2 is strictly positive) and (see Eq. (43)) ${}_0\xi^1_3 \neq 0$, we can fit ${}_1\xi^1_3$ such that Eq. (47)

be satisfied too. Furthermore, it can be seen (see Appendix A) that, to get $P'^3 = 0$, ψ can always be fitted such that Eq. (48) becomes satisfied.

Next, we consider the three remaining Eqs. (49)-(50). Since (see again (43)) ${}_1\xi^0 \neq 0$ we can fit ${}_0\xi^2$ such as to have (50). Similarly for Eq. (51) by fitting ${}_1\xi^1$. Finally, it can be proved (see Appendix B) that the Jacobian matrix (45) can always be fitted in order to have Eq. (49) satisfied.

In all, we have just proved that for any *universe* there always exist intrinsic coordinate systems, that is Gaussian coordinates, $\{x'^\alpha\}$, satisfying the supplementary conditions (39), and such that $P'^i = 0$ and, irrespective of the angular momentum origin, $J'^{ij} = 0$.

4 Creatable universes

Let it be a *universe* that we have referred to intrinsic coordinates $\{x'^\alpha\}$. Then, we will call that *universe* a *creatable universe* if in these coordinates we also have:

$$P'^0 = 0, \quad J'^{0i} = 0. \tag{52}$$

This means, according to Eqs. (8), (12) and (13), that

$$P'^0 = -\kappa \int {}_0g'^1_{aa} dx'^1 dx'^2 = 0, \tag{53}$$

$$J'^{0a} = \kappa \int x'^a \; {}_0g'^1_{bb} dx'^1 dx'^2 = 0, \tag{54}$$

$$J'^{03} = -\kappa \int {}_0g'^0_{aa} dx'^1 dx'^2 = -2\kappa \int f' dx'^1 dx'^2 = 0. \tag{55}$$

that is, ${}_0g'^1_{aa}$ and f' must be such that the above four integrals vanish.

On the other hand, we find after some calculation

$${}_0g'^1_{aa} = \;({}_1g^0_{bc}\;{}_0\xi^1 + \;{}_0g^1_{bc}\;{}_0\xi^1_3) M_{ba} M_{ca} = \lambda^2 ({}_1g^0_{aa}\;{}_0\xi^1 + \;{}_0g^1_{aa}\;{}_0\xi^1_3) \tag{56}$$

which can be compared with (48). Notice that here we are left with no more freedom to fit a given value of ${}_0g'^1_{aa}$ in order to have (53) and (54): in fact, both, the Jacobian matrix M_{ab}, plus ${}_0\xi^1$ and ${}_0\xi^1_3$ (that is to say, plus ψ, according to (43) and (44)), have already been fitted such as to have intrinsic coordinates. This means, that:

A *universe* is not necessarily a creatable universe, which even if expected is a very remarkable result.

Now, before we can continue, we must say something about Eq. (55), that would have to be satisfied if, according to our definition, we have a creatable universe. Since f' is strictly positive it seems at first sight that (55) can only be satisfied in any one of the two following cases: first, if the area of Σ_2 vanishes (in which case f' should remain conveniently bounded when we approach Σ_2; notice that the boundary Σ_2 could not belong to Σ'_3, in which case f' could go to infinite when we approach Σ_2); second, if f' goes to zero when we approach Σ'_2, which means again that Σ_2 does not belong to Σ'_3.

But, actually, these are not the only cases where we can have (55), since, according to what is said at the end of section 2, Σ_2 could have several different sheets, and it could happen that the different contributions from these different sheets compensate among them to give a vanishing value for $\int f' dx'^1 dx'^2$. Thus, in Minkowski space, M_4, in Lorentzian coordinates (which are *intrinsic* coordinates) we have $f' = 1$. But, Σ_2 is made from six sheets, the six faces of a cube that increases without limit. Then, the two contributions corresponding to two opposite faces cancel each one to the other.

Anywise, some one could argue that we could only define a given *universe* as a creatable universe if $P^\alpha = J^{\alpha\beta} = 0$ for ANY intrinsic coordinate system. But this would be an exceeding demand since not even the case of the Minkowski space-time, M_4, would satisfy such a strong requirement. Actually, one type of intrinsic coordinates for this *universe* are the standard Lorentz coordinates. Furthermore, in these coordinates, all 4-momenta, P^α and $J^{\alpha\beta}$ vanish, so that this *universe* is a creatable universe according to the definition we have just given. Nevertheless, it can be easy seen (see Appendix C) that starting from Lorentz coordinates, one can always make an elementary coordinate transformation leading to new, non Lorentzian, intrinsic coordinates, such that the new energy P'^0 does no more vanish. Obviously, according to section 3, this elementary coordinate transformation has to be one where the infinitesimal version of the coefficients ${}_0\xi^0$ and ${}_0\xi^0_3$ do not vanish, that is Eq. (15) does not more occur.

The reason for this non vanishing energy, P'^0, in M_4 is that, by doing the above elementary coordinate transformation, we have left a coordinate system (the Lorentzian one) which was well adapted to the symmetries of the Minkowskian metric: the ones tied to the ten parameters of the Poincaré group.

Thus, given a *universe* which has $P^\alpha = J^{\alpha\beta} = 0$ for some intrinsic coordinate system, if there are other intrinsic coordinates where this vanishing is not preserved, we should consider that this non preservation expresses the fact that the new intrinsic coordinates are not well adapted to some basic metric symmetries. To which symmetries, to be more precise? In general terms, to the ones which allow us to have just vanishing linear and angular 4-momenta for some intrinsic coordinate system.

In other words: in spite of the apparent freedom in the choice of the coordinate frame, we have characterized in an intrinsic way if a *universe* has or has not vanishing 4-momenta. In our framework, in order to have this vanishing, we only need to find ONE *intrinsic* coordinate frame where P^α and $J^{\alpha\beta}$ vanish, which, as we have just explained, can only been found in some special *universes.*

5 The perturbed FLRW universes

In Ref. [25] the creatibility of perturbed FLRW universes was addressed. The main result of that paper which concerns us here is that in the flat case it is

found that the energy is infinite, $P^0 = \infty$, for inflationary scalar perturbations plus arbitrary tensor perturbations. This seems to say that inflationary perturbed flat FLRW universes are not creatable. Nevertheless, as it has been already stressed at the end of the Introduction, this assessment needs to be validated in the new framework we have developed in the present paper, where creatibility can only be considered for intrinsic coordinate systems, i. e., systems where, in particular, the linear and angular 3-momenta, P^i and J^{ij}, vanish.

Then, we prove next that both momenta vanish in the coordinate system where it was obtained that $P^0 = \infty$. Therefore, we conclude that, in the new framework of the present paper, the non creatibility of the inflationary perturbed flat FLRW universe remains unchanged.

Let us prove first that P^i vanish. According to Ref. [25] we write the perturbed 3-space metric dl^2 as

$$dl^2 = \frac{a^2(t)}{(1+\frac{\mathrm{k}}{4}r^2)^2}(\delta_{ij}+h_{ij})dx^i dx^j, \tag{57}$$

where $a(t)$ is the cosmic expansion factor.

In the flat case, $\mathrm{k}=0$, when considering inflationary scalar perturbations, the perturbed 3-space metric, h_{ij}, reads

$$h_{ij}(\mathbf{x},\tau) = \int \exp(i\mathbf{k}\cdot\mathbf{x}) h_{ij}(\mathbf{k},\tau) d^3k \tag{58}$$

with the following expression for the Fourier transformed function $h_{ij}(\mathbf{k},\tau)$:

$$h_{ij}(\mathbf{k},\tau) = h(\mathbf{k},\tau)\hat{k}_i\hat{k}_j + 6\eta(\mathbf{x},\tau)(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}). \tag{59}$$

Here $h \equiv h_{kk}$ and η are convenient functions, $\hat{k}_i \equiv k_i/k$, $k \equiv \sqrt{k_i k^i}$, and τ is defined such that $dt/d\tau \equiv a$.

According to Eq. (2):

$$P^i = \lim_{r\to\infty} \frac{r^2}{16\pi G}\int I^i d^3k \tag{60}$$

where

$$\begin{aligned} I^i &\equiv \int \exp(i\mathbf{k}\cdot\mathbf{x})[\dot{h}_{kk}(\mathbf{k},\tau)\delta_{ij} - \dot{h}_{ij}(\mathbf{k},\tau)]n_j d\Omega \\ &= \int \exp(i\mathbf{k}\cdot\mathbf{x})[\dot{h}(\mathbf{k},\tau)(\delta_{ij}-\hat{k}_i\hat{k}_j) + 6\dot{\eta}(\mathbf{x},\tau)(\frac{1}{3}\delta_{ij}-\hat{k}_i\hat{k}_j)]n_j d\Omega. \end{aligned} \tag{61}$$

Here, the dot stands for the time, t, derivative and with $d\Omega$ the integration element of solid angle.

Notice that here we have taken as Σ_2 the 2-surface $t=t_0$, $r=R\to\infty$, instead of the six faces of the over growing cube reported at the end of section 2. Of course, the 3-volume element remains $dx^1dx^2dx^3$ since these are the corresponding intrinsic coordinates. We can take $r=R\to\infty$ for Σ_2 because

of the choice of the above cube had only the function of making easier the proof of the existence of intrinsic coordinates.

On the other hand, one easily finds

$$\int \exp(i\mathbf{k}\cdot\mathbf{x})n_i d\Omega = \frac{4\pi i}{kr}(\frac{\sin kr}{kr} - \cos kr)\hat{k}_i \equiv \Phi(k,r)\hat{k}_i \tag{62}$$

where what is important for us here is that Φ does not depend on $\hat{k}_i$. Then

$$I^i = \Phi[6\dot{\eta}(\mathbf{x},\tau)(\frac{1}{3}\hat{k}_i - \hat{k}_i)] = -4\Phi\dot{\eta}(\mathbf{x},\tau)\hat{k}_i. \tag{63}$$

But, as it has been quoted in [25], in the case of inflationary scalar perturbations, in which we are interested here, $\eta(\mathbf{k},\tau)$ does not actually depend on $\hat{\mathbf{k}}$. Then, by symmetry, $\int I^i d^3k = 0$, and so, $P^i = 0$ for any time.

Next, we consider general tensor perturbations and we see that P^i vanish too. As quoted again in Ref. [25], the above Fourier transformed function $h_{ij}(\mathbf{k},\tau)$ reads now:

$$h_{ij}(\mathbf{k},\tau) = H(k,\tau)\epsilon_{ij}(\hat{k}), \tag{64}$$

where the symmetric matrix ϵ_{ij} is transverse and traceless:

$$\epsilon_{ij}k_i = 0, \quad \epsilon_{ii} = 0. \tag{65}$$

The above I^i integral becomes now

$$I^i = -\int \exp(i\mathbf{k}\cdot\mathbf{x})H(k,\tau)\epsilon_{ij}n_j d\Omega, \tag{66}$$

which according to (62) and the first equation in (65) becomes $I^i = 0$. This is, we have again $P^i = 0$.

Thus, when inflationary scalar and general tensor perturbations are both present we have $P^i = 0$, as we wanted to prove.

The next step will be to prove that, for any time, J^{jk} vanish too for both types of perturbations. Let us first consider inflationary scalar perturbations, that is, Eq. (59).

According to Eq. (11):

$$J^{jk} = \lim_{r\to\infty}\frac{r^3}{16\pi G}\int I^{jk}d^3k, \tag{67}$$

where

$$I^{jk} = \int \exp(i\mathbf{k}\cdot\mathbf{x})[n_k\dot{h}_{ij}(\mathbf{k},\tau) - n_j\dot{h}_{ki}(\mathbf{k},\tau)]n_i d\Omega. \tag{68}$$

But, obviously:

$$\int \exp(i\mathbf{k}\cdot\mathbf{x})n_i n_j d\Omega \propto \delta_{ij},\ \hat{k}_i\hat{k}_j, \tag{69}$$

that is, the calculation of this integral must give a contribution which goes like δ_{ij}, and another one which goes like $\hat{k}_i\hat{k}_j$. Then, it is easy to verify that when

these two kinds of contributions are introduced in (68) we obtain identically $I^{jk} = 0$, and so $J^{jk} = 0$.

Finally, we will consider general tensor perturbations, that is, $h_{ij}(\mathbf{k}, \tau)$ given by Eqs. (64) and (65). In this case (68) becomes

$$I^{jk} = \dot{H}(k, \tau) \int \exp(i\mathbf{k} \cdot \mathbf{x})(n_k \epsilon_{ij} - n_j \epsilon_{ki}) n_i d\Omega. \tag{70}$$

But having in mind (69) and the first equation of (65) it is straightforward to see that I^{jk} and then J^{jk} vanish.

All in all, for any time, P^i and J^{ij} vanish in the same coordinate system where it was proved (see Ref. [25]) that $P^0 = +\infty$. Then, we can assert that our perturbed flat FLRW universe is really a non creatable one.

On the other hand, it can be easily seen that in the present new approach, as in [25], perturbed closed FLRW universes are creatable, while perturbed open FLRW universes are not.

6 Final considerations

The energy of Friedmann-Lemaître-Robertson-Walker (FLRW) cosmologies has been calculated by different authors using divers procedures, like pseudotensorial methods based on specific choices of coordinates [29,30,31,32,33], or Hamiltonian methods imposing boundary conditions [13,21], or by choosing an appropriate background configuration [2,8], or even by other procedures [10]. Quasi-local approaches have also been extensively considered, providing distinct results because of the different used definitions [9,34]. Many authors (us including) agree with the following statement: the total energy vanishes both for closed and flat FLRW universes, but diverges to $-\infty$ for the class of open models (negative curvature index, $k = -1$). Thus, the closed and flat FLRW universes would be creatable, but the open one would be not.

However, there is no full agreement in the current literature on these energy values (cf [31,35]) although, in our opinion, their goodness becomes supported by the rightness of the criteria we have implemented in the present paper to define proper values for all 4-momenta components. Among the references which agree with these FLRW values are [10,29]

Let us specify that these values were obtained by us in [15], but the translation of the result from the old framework in [15] to the new one in the present paper is straightforward.

Notice that the same conclusion follows from the results obtained in [2] concerning integral conservation laws with respect to a given background and its associated isometry group, but only when this background is the flat spacetime.

On the other hand, the creatibility of the perturbed FLRW universes (see Section 5) should be also analyzed following the approach of Ref. [2]. In this case, the above conclusion about the non-perturbed case strongly suggests that the results presented in Sect. 5 could be recovered from the results of

[2] under these assumptions: (i) the considered background is the Minkowski space-time, (ii) the conservation laws are referred to the background isometries, and (iii) the perturbed metric and the energy content are considered in some synchronous gauge (by taking Gauss coordinates).

Now, before ending the paper we would like to point out that the main interest of it could be to give a criterion to discard from the very beginning as much as possible space-times as candidates to represent our actual Universe. The criterion could be that good initial candidates must be *creatable* universes. Thus, as commented above, in [25] it was claimed that, within the inflationary perturbed FLRW universes, only the closed case corresponds to a *creatable* universe. Of course, the criterion is not a consequence of the theory of the General Relativity when applied to cosmology. It is only a guess, one appearing in the literature at last since 1973 [23,24] that we find so appealing, in our opinion, as to deserve that its consequences be explored, as we have continued to do in the present paper. This result, obtained in [25] in a non conclusive way, has been fully validated in the framework of the present paper, as it has been proved in Sect. 5. Similarly, since some other space-times have lately been considered as candidates to represent our Universe (see for example, [36], [37]), we could check them to see if they fulfill the above criterion of creatibility. When making this checking, in the case we obtained $P^\alpha = 0$ and $J^{\alpha\beta} = 0$ for a given $t = t_0$, we still had to verify that the result does not depend of the value of t_0, that is, we would have to verify *a posteriori* that we were dealing with a space-time which is a *universe*. All this would deserve some future work.

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A Fitting the function ψ to get $P'^3 = 0$

We must fit ψ such that ${}_1g'^0_{aa}$, given by (see (48))

$$ {}_1g'^0_{aa} = \lambda^2({}_1g^0_{aa}\ {}_1\xi^0 + \ {}_0g^1_{aa}\ {}_1\xi^0_3)\,, \tag{71} $$

gives $P'^3 = 0$. Notice that according to Eq. (10) we have

$$ P'^3 = \kappa \int {}_1g'^0_{aa}\ dx'^1 dx'^2\,. \tag{72} $$

On the other hand, from (43) and (44), the equation (71) can be written as

$$ a = b\cosh\psi + c\sinh\psi\,, \tag{73} $$

where

$$ a \equiv\ {}_1g'^0_{aa}, \quad b \equiv \lambda^2\ {}_1g^0_{aa}, \quad c \equiv \frac{\lambda^2}{\sqrt{f}}\ {}_0g^1_{aa} \tag{74} $$

Then, putting $\cosh\psi \equiv x$, we obtain the algebraic second order equation

$$ (b^2 - c^2)x^2 - 2abx + a^2 + c^2 = 0\,, \tag{75} $$

that only has real solutions if

$$a^2 + c^2 \geq b^2 \,. \tag{76}$$

But we can ensure it by taking a large enough. This can always be made since if $a \equiv {}_1g'^0_{aa} \neq 0$ is such that $\int a \; dx'^1 dx'^2 = 0$, then we also will have $\int K a dx'^1 dx'^2 = 0$, with K a constant whose absolute value, $|K|$, is as large as we wanted.[3] Furthermore, if $|K|$ is large enough, we can easily see that for the new coefficient a, that is, for Ka, one at least of the x solutions is larger than one, as it must be.

B Fitting conveniently the functions $\boldsymbol{\lambda}$ and $\boldsymbol{\theta}$ or the functions $\boldsymbol{\lambda}$ and $\boldsymbol{\psi}$

According to what is said at the end of section 3, we must fit the functions λ and θ such that Eq. (49) be satisfied. Taking in account (46), the Eq. (49) becomes:

$${}_1g'^0_{3a} = (\,{}_1\xi^0\;{}_0\xi^1_3 - {}_0\xi^1\;{}_1\xi^0_3)\,{}_1g^0_{3b}M_{ba} + f(\,{}_0\xi^1_3\;{}_1\xi^0_{3,a} - {}_1\xi^0_3\;{}_0\xi^1_{3,a}) + {}_1\xi^0\;{}_0\xi^1_{,a} - {}_0\xi^1\;{}_1\xi^0_{,a} \tag{77}$$

where ${}_1\xi^0_{3,a} \equiv \frac{\partial\;{}_1\xi^0_3}{\partial x'^a}$, and so on. Furthermore, having in mind (43), (44) and the definition of λ in (45), Eq. (77) becomes:

$${}_1g'^0_{3a} = \lambda(M_{ba}\;{}_1g^0_{3b} + X_a), \tag{78}$$

where we have put

$$X_a \equiv \frac{2}{\sqrt{f}}\;\frac{\partial\psi}{\partial x'^a}. \tag{79}$$

Then, from (45), we obtain the system

$$\lambda^2(\,{}_1g^0_{31}\cos\theta - \;{}_1g^0_{32}\sin\theta) = -\lambda X_1 + \;{}_1g'^0_{31} \tag{80}$$

$$\lambda^2(\,{}_1g^0_{32}\cos\theta + \;{}_1g^0_{31}\sin\theta) = -\lambda X_2 + \;{}_1g'^0_{32}. \tag{81}$$

Notice that, in this system, the functions ${}_1g'^0_{3a}$ are defined modulus an arbitrary constant factor K (as it was, above, the case with ${}_1g'^0_{aa}$). This means that, in (80) and (81), we can take ${}_1g'^0_{3a}$ as small as we want, provided that the original ${}_1g'^0_{3a}$ remain bounded (the unbounded special case will be considered next), which in turn means that we can take as the system to solve

$$\lambda(\,{}_1g^0_{31}\cos\theta - \;{}_1g^0_{32}\sin\theta) = -X_1 \tag{82}$$

$$\lambda(\,{}_1g^0_{32}\cos\theta + \;{}_1g^0_{31}\sin\theta) = -X_2, \tag{83}$$

whose unique solution, out of the singular case ${}_1g^0_{3a} = 0$, is

$$\lambda\cos\theta = -\frac{{}_1g^0_{31}X_1 + \;{}_1g^0_{32}X_2}{(\,{}_1g^0_{31})^2 + (\,{}_1g^0_{32})^2} \equiv Y_1, \tag{84}$$

$$\lambda\sin\theta = \frac{{}_1g^0_{32}X_1 - \;{}_1g^0_{31}X_2}{(\,{}_1g^0_{31})^2 + (\,{}_1g^0_{32})^2} \equiv Y_2, \tag{85}$$

that is to say

$$\lambda = \sqrt{Y_1^2 + Y_2^2}, \quad \tan\theta = \frac{Y_2}{Y_1}. \tag{86}$$

---

[3] The singular case $a \equiv \;{}_1g'^0_{aa} = 0$, would give as a solution for (73) $\tanh\psi = -b/c$, which only exists if $|b/c| < 1$.

To complete the above discussion let us consider the special case where ${}_1g'^0_{3a}$ goes to infinite when we approach Σ_2. (Obviously this will have to be compatible with the vanishing of the integrals $\int {}_1g'^0_{3a} dx'^1 dx'^2$). In this case, the system (80), (81), becomes:

$$\lambda^2({}_1g^0_{31} \cos\theta - {}_1g^0_{32} \sin\theta) = {}_1g'^0_{31} \tag{87}$$
$$\lambda^2({}_1g^0_{32} \cos\theta + {}_1g^0_{31} \sin\theta) = {}_1g'^0_{32}, \tag{88}$$

with ${}_1g'^0_{3a}$ going to infinite, whose solution is

$$\lambda^2 = \infty, \tan\theta = \lim_{{}_1g'^0_{3a} \to \infty} \frac{{}_1g^0_{31}\ {}_1g'^0_{32} - {}_1g^0_{32}\ {}_1g'^0_{31}}{{}_1g^0_{31}\ {}_1g'^0_{31} + {}_1g^0_{32}\ {}_1g'^0_{32}}. \tag{89}$$

We could still consider the remaining two special cases where, only one of the two functions ${}_1g'^0_{3a}$ goes to infinite, but the reader can see easily than also in both cases a solution exists for λ, θ.

To end with this Appendix B, let us consider the above singular case ${}_1g^0_{3a} = 0$. It seems that now the four Eqs. (46) and (49) cannot always be satisfied by fitting ${}_1\xi^1_b$ and M_{ab} since these four unknown functions appear now through only two quantities ${}_1\xi^1_b M_{ba}$.

Nevertheless, let us proceed along the following lines:

As far as Eq. (49) is concerned, we always can satisfy it by fitting some convenient values of ${}_1\xi^1_b$, since $f \neq 0$ and M_{ab} is a regular matrix.

On the other hand, according to (78) and (79), Eq. (46) reads now

$${}_1g'^0_{3a} = \frac{2\lambda}{\sqrt{f}} \frac{\partial\psi}{\partial x'^a}. \tag{90}$$

Using λ as an integrating factor, we always can find a family of solutions ψ of these two equations. Then, we must fit this family of solutions such that the Eq. (48) we are left with,

$${}_1g'^0_{aa} = \lambda^2({}_1g^0_{aa} \cosh\psi + \frac{{}_0g^1_{aa}}{\sqrt{f}} \sinh\psi), \tag{91}$$

becomes satisfied. To see that this is also possible, in (90) we will choose ${}_1g'^0_{3a} = \epsilon_a g_3$, with $\epsilon_a = 1$, $\forall a$, and g_3 a function such that $\int g_3 dx'^1 dx'^2 = 0$. In this case we have $\frac{\partial\psi}{\partial x'^1} = \frac{\partial\psi}{\partial x'^2}$, that is ψ is a function of $x'^1 + x'^2 \equiv y_1$, but not of $y_2 \equiv x'^1 - x'^2$:

$$\frac{\partial\psi}{\partial y_2} = 0. \tag{92}$$

Then, let us integrate (91) along y_2 over Σ_2. We will have

$$a = b\cosh\psi + c\sinh\psi\,, \tag{93}$$

with

$$a = \int {}_1g'^0_{aa} dy_2,\ b = \int \lambda^2\ {}_1g^0_{aa} dy_2,\ c = \int \frac{\lambda^2}{\sqrt{f}}\ {}_0g^1_{aa} dy_2, \tag{94}$$

where, like ψ, the coefficients a, b, c, depend only on y_1. On the ground of what was said for the coefficient a of Appendix A, the present coefficient a is also as greater as we want. Then, we can conclude that (93) always have a solution for ψ for any function ${}_1g'^0_{aa}$ such that $\int {}_1g'^0_{aa} dx'^1 dx'^2 = 0$. That is to say, Eqs. (46), (48) and (49) can all be satisfied at the same time, as we wanted to prove in the present singular case ${}_1g^0_{3a} = 0$.

C The counter example of Minkowski space

In section 4, we claim that if we have a *universe* such that its ten 4-momenta vanish for some given intrinsic system of coordinates, we cannot hope to keep this ten-fold vanishing against any coordinate change going to new intrinsic coordinates. The reason of this is that even Minkowski space, M_4, have not such a property.

In order to see this, refer M_4 to Lorentzian coordinates. These are obviously intrinsic coordinates, in the sense of the present paper. Furthermore, all ten 4-momenta vanish in this Lorentzian frame. Thus, according to our definition, M_4 is an example of creatable universe. Then, let us make some general infinitesimal coordinate transformation:

$$x^\alpha = x'^\alpha + \epsilon^\alpha(x)\,, \tag{95}$$

where the old coordinates, $\{x^\alpha\}$, are Lorentzian coordinates. Let us subject the functions $\epsilon(x)$ to the condition that the new coordinates $\{x'^\alpha\}$ be intrinsic coordinates. That is, the new metric components

$$g'_{\alpha\beta} = \eta_{\alpha\beta} + \eta_{\alpha\rho}\partial_\beta\epsilon^\rho + \eta_{\beta\rho}\partial_\alpha\epsilon^\rho \tag{96}$$

has to satisfy on the one hand, Eqs. (25) and (26) (the first one up to zero order in $t' - t_0$ and order one in x'^3). On the other hand, the time derivatives $\partial'_0 g'_{3i}$, $\partial'_0 g'_{aa}$, must fulfill the conditions (34)-(36)

$$\int {}_1g'^0_{aa}\ dx^1dx^2 = 0,\ \int {}_1g'^0_{3i}\ dx^1dx^2 = 0, \tag{97}$$

$$\int (x'^2\ {}_1g'^0_{31} - x'^1\ {}_1g'^0_{32})\ dx^1dx^2 = 0, \tag{98}$$

$$\int x'^a\ {}_1g'^0_{33}\ dx^1dx^2 = 0, \tag{99}$$

which mean that $P'^i = 0$ and that, irrespective of the origin of the angular momentum, $J'^{ij} = 0$ (notice that to first order we can put dx^1dx^2 instead of $dx'^1dx'^2$).

After some elementary calculations, all these conditions are written:

$$ {}_1\varepsilon^0_a =\ \partial_a\ {}_0\varepsilon^0, \quad {}_1\varepsilon^0_3 =\ {}_0\varepsilon^1, \quad {}_1\varepsilon^0 = 0, \tag{100}$$

$$ {}_1\varepsilon^1_a = \partial_a\ {}_0\varepsilon^1, \quad {}_1\varepsilon^1_3 =\ {}_0\varepsilon^2, \tag{101}$$

$$ {}_0\varepsilon^1_a =\ -\partial_a\ {}_0\varepsilon^0_3, \quad {}_0\varepsilon^1_3 = (1-f')/2, \tag{102}$$

$$ {}_1g'^0_{3a} = \partial_a\ {}_1\varepsilon^0_3 +\ {}_1\varepsilon^1_a,\ {}_1g'^0_{33} = 2\ {}_1\varepsilon^1_3,\ \ {}_1g'^0_{aa} = 2\partial_a\ {}_1\varepsilon^0_a, \tag{103}$$

where we have used the notation $\varepsilon^i \equiv \varepsilon_i$.

A particular solution of this system is

$$ {}_0\varepsilon^1 =\ {}_1\varepsilon^0 = 0,\ \ {}_1\varepsilon^0_i =\ {}_1\varepsilon^1_i = 0,\ {}_0\varepsilon^1_a = -\partial_a\ {}_0\varepsilon^0_3, \tag{104}$$

$$ {}_0\varepsilon^1_3 = (1-f')/2, \quad \partial^2_{aa}\ {}_0\varepsilon^0 = 0. \tag{105}$$

On the other hand, we similarly obtain:

$$ {}_0g'^1_{aa} = 2\partial_a\ {}_0\varepsilon^1_a \tag{106}$$

which, according to the corresponding equation in (104), becomes

$$ {}_0g'^1_{aa} = -\partial^2_{aa}\ {}_0\varepsilon^0_3. \tag{107}$$

Thus, since ${}_0\varepsilon^0_3$ is small, but otherwise arbitrary, we always can choose ${}_0\varepsilon^0_3$ so as to have

$$\int {}_0g'^1_{aa}\ dx^1dx^2 \neq 0, \tag{108}$$

that is, so as to have $P'^0 \neq 0$. Then, as we have announced, we cannot preserve the vanishing of P'^α and $J'^{\alpha\beta}$ when making a general coordinate transformation from an intrinsic coordinate system to another intrinsic one.

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